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**Fast on-line determination of quasi-optimal  
grasp configurations based on off-line analysis  
and parametrization on the wrist position**

**Gerrit Färber, Raúl Suárez**

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Institut d'Organització i Control  
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## *— Abstract —*

Determining the optimum configuration of a mechanical hand in order to grasp an object is a computational hard work, mainly due to the large number of degrees of freedom (fingers and wrist), the existence of a large number of solutions, and the constraints imposed by the object or the task to be done. The paper presents an approach to solve this problem under certain conditions, which is then particularized for an anthropomorphic mechanical hand involving 22 degrees of freedom. The configurations of the hand to optimally grasp rectangular parallelepipeds of different sizes with a planar grasp (contact points on the same plane) are determined off-line from the results the wrist position is analyzed and stored as approximated functions of the height and width of the grasped object. This information is used for a fast on-line computation of the hand configuration in a real grasp operation, given the rectangular parallelepiped bounding-box of the part of the object that will be "inside" the hand after the grasp. The approach has been implemented for the mechanical hand MA-I and the paper includes numerical examples.

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## **I. INTRODUCTION**

The grasp of an object, performed by a mechanical hand, is subject to several considerations like dexterity, equilibrium, stability and dynamic behavior [1]. The quality of the grasp regarding these topics depends on the grasp configuration of the mechanical hand. The use of anthropomorphic mechanical hands has the advantage of allowing redundant solutions, this, however, has the drawback that infinite solutions are possible and the determination of a “good”, if not “optimal”, solution is a computational hard work.

Although there are works that, in one way or another, consider the hand kinematics in the grasp computation (for instance [2], [3]), quite frequently, the grasping problem is addressed focused only on the object to be grasped without considering the mechanical hand (for instance [4], [5], [6], [7], among several others). This may result in an excellent set of contact points for the fingers, ensuring a robust grasp,

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but due to the limitations imposed by the kinematics of a given hand, the grasp may be impossible to be performed. Other works deal with the problem of positioning the hand given the set of grasping contact points, which is equivalent to the inverse kinematics problem. Nevertheless, since the hand kinematics was not considered in the search of the grasp contact points there may be long iterative processes generating, first, a set of contact points for the fingers and, then, trying to find an actual hand configuration to perform the grasp. This may be a time consuming approach, so learning approaches were proposed to do it faster during a real task execution. As an example, the use of neural nets is presented in [8], but strong assumptions are considered to provide robustness to the system, like for instance that the initial orientation of the wrist in the solution search is smaller than  $90^\circ$  (so the hand is already “well” oriented) and the position search space for the wrist position is relatively small. Finding a proper wrist position and orientation is a key point because with this information solving the inverse kinematics of an anthropomorphic mechanical hand is relatively simple.

In this context, the work in this paper proposes a new approach to quickly generate a hand configuration, close to the optimum, during on-line real operation. As in the traditional learning approaches, a computational hard work is done off-line, but the difference is that in this approach the knowledge is captured as explicit functions that provide the position of the wrist given the height and width of the object to be grasped. With this information, the online-determination of the whole hand configuration can be done in a simple way.

The approach was implemented for the anthropomorphic mechanical hand MA-I developed and built at the Robotics Lab of the IOC. Numerical examples are included in the paper.

## II. ASSUMPTIONS AND APPROACH OVERVIEW

The following assumptions are initially considered in this work (some of them can be easily relaxed to obtain variations of the approach):

The contacts between the fingers and the object is a point (the called “hard contact” in [4]), but the fingers can contact the object in different points of the fingertips (note that in the general case this adds 2 “virtual” degrees of freedom per finger to the problem). The fingertips are assumed to be spherical.

The hand is anthropomorphic and will perform the most common planar grasp, so the contact points are in the same plane and the thumb will push against the other fingers. For simplicity and without loss of generality, the grasp plane is assumed to be horizontal.

The object is considered to be a rectangular parallelepiped, which in any case is a simple bounding box for a real object. This consideration is relevant because the grasp must avoid the collision of the object with the palm and the fingers of the hand. Figure 1 illustrates these considerations showing a simplified schematic of the hand grasping the object. Finally, the optimization criterion for the hand configuration is that all the joints of the hand are as close to the middle of their ranges as possible.

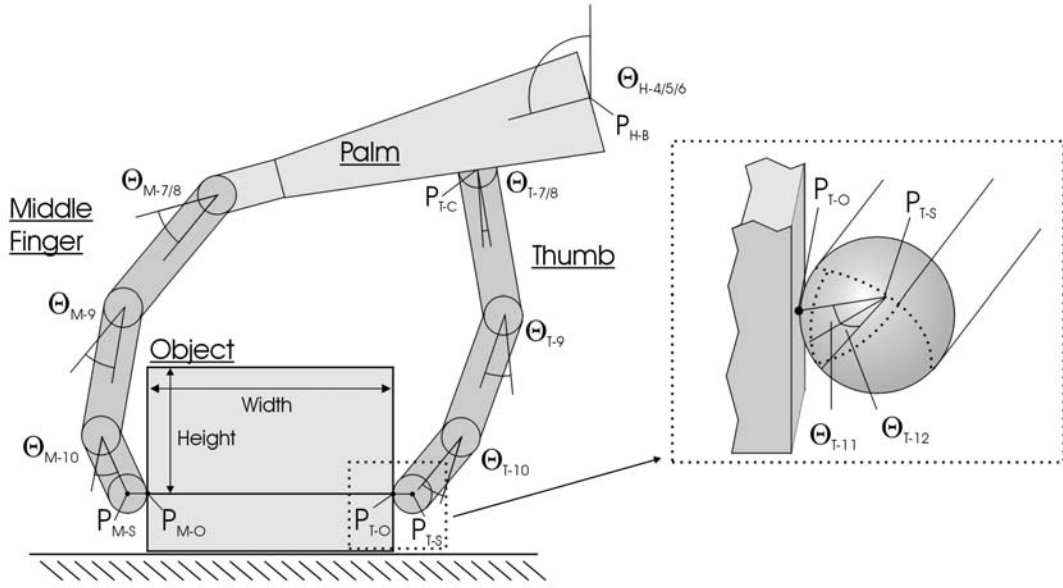


Figure 1. Schematic of the hand grasping an object defined by its height and width (only two fingers are shown). The inset shows the 2 virtual articulations in the fingertips.

In order to provide an overview of the proposed approach we will describe it according to the following steps.

*Steps to be done off-line:*

1. Determination of the optimum configuration of the hand (wrist and fingers) for objects of different height and width. This is done once for a finite number of pairs [height, width] using a search procedure. The starting position of the wrist is determined with a heuristics in order to accelerate the process. The search tries to minimize the distance from the joint positions to the middle of their ranges.
2. Analysis of the wrist positions, which are approximated with simple functions of the object's height and width. The  $X$ ,  $Y$  and  $Z$ -coordinates of the wrist positions are then approximated with a few second order surfaces over proper regions in the domain defined by the variables [height, width]. These functions are stored for their on-line use.

*Steps to be done on-line:*

1. Determination of the object size (or proper bounding box) determining the length of the object that the grasp must be able to accept inside the hand (object height), and the distance between the main planes where the fingers will touch (object width).
2. Identification of the domain containing the pair [height, width] corresponding to the actual object and apply the three corresponding second order functions to obtain the  $X$ ,  $Y$  and  $Z$ -coordinates of the wrist position.
3. Solution of the inverse kinematics of the fingers and the orientation of the palm for the wrist position obtained in the previous step.

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This approach presents a good balance between the amount of information that has to be stored and the amount of computation that has to be done on-line, being both of them relatively small and quite acceptable. Moreover, from the research point of view, one particular advantage of the approach is that the information obtained off-line explicitly shows the behavior of the hand when an optimal grasp is performed, allowing a better understanding of the grasp action and opening new opportunities for further improvements. This is not possible with other techniques, where the knowledge is implicitly stored, like, for instance, in neural nets.

The rest of the paper deals with the off-line steps of the approach, using the anthropomorphic mechanical hand MA-I (described below) to introduce basic nomenclature and facilitate explanations. The on-line steps are straightforward and therefore are not discussed in the paper.

### III. DESCRIPTION OF THE HAND

The hand MA-I [9] consists of a palm and four fingers: thumb, index-, middle- and ring-finger, and it is assembled with the wrist of a robot arm with six degrees of freedom (d.o.f.) (Figure 2).

The following basic nomenclature for the subindices is used along the paper:

$H$	Hand
$F$	Finger (to be specified as: $T, I, M, R$ )
$T$	Thumb
$I$	Index finger
$M$	Middle finger
$R$	Ring finger

Furthermore, the following nomenclature is also used (see Figure 1) (positions are defined in a world reference system unless indicated differently):

$P_{F-O}$	Position of contact point finger-object
$P_{F-S}^i$	Position of the base of articulation $i$ of finger $F$
$P_{F-S}$	Position of the centre of the sphere in the last phalanx (fingertip) of finger $F$ ( $P_{F-S}^{I0}$ )
$P_{F-C}$	Position of the connection finger-palm
$P_{H-B}$	Position of the hand-base
$\Theta_{F-i}$	Angle of articulation $i$ of Finger $F$
$D_{TS-MS}$	Distance between $P_{T-S}$ and $P_{M-S}$ .
$D_{TC-MS}$	Distance between $P_{T-C}$ and $P_{M-S}$ .
$D_{TC-TS}$	Distance between $P_{T-C}$ and $P_{T-S}$ .
$X_F, Y_F, Z_F$	Position of $P_{F-S}$ with $\Theta_{H-4}$ to $\Theta_{H-6}$ equal to 0. ( Local position of $P_{F-S}$ with respect to $P_{H-B}$ )

In order for the explanation to be independent of the arm manipulator, it will be considered that the position,  $P_{H-B}$ , of the hand is determined by three parameters  $X_{H-B}$ ,  $Y_{H-B}$  and  $Z_{H-B}$  and the orientation of the hand is determined by three virtual joints  $\Theta_{H-4}$ ,  $\Theta_{H-5}$  and  $\Theta_{H-6}$ . The values of  $X_{H-B}$ ,  $Y_{H-B}$ ,  $Z_{H-B}$  and  $\Theta_{H-4}$ ,  $\Theta_{H-5}$  and  $\Theta_{H-6}$  describe the configuration of the robot wrist and are obtained from the six real joints of the robot arm. The articulations  $\Theta_{F-7}$  to  $\Theta_{F-10}$  define the angles of the rigid

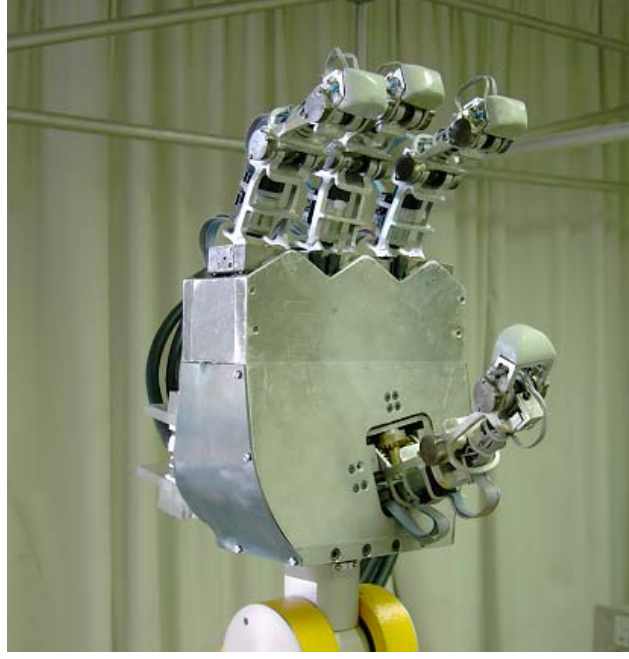


Figure 2. Mechanical Hand MA-I.

phalanges of the fingers ( $\Theta_{F-7}$  being the abduction-adduction d.o.f.). The articulations  $\Theta_{F-11}$  and  $\Theta_{F-12}$  are virtual articulations that describe the position of the contact point on the spherical fingertip of each finger (Figure 1).

The kinematics chain from  $P_{T-O}$  to  $P_{M-O}$  can be divided into two individual chains. The first chain is formed by the middle finger together with the palm and the second chain is formed by the thumb. The common point of the two chains is  $P_{T-C}$ , where the thumb is connected with the palm.  $P_{T-C}$  serves as an auxiliary reference-point of the hand.

#### IV. DETAILED PROCEDURE

The detailed description of the proposed off-line procedure is done according to the following steps:

1. Determination of the optimum configuration of the hand (wrist and fingers) for objects of different height and width. For each pair [height, width] within the desired range (in our case: 200x200 mm) the following actions are done.
  - 1.1. Determination of the optimum configuration of the hand considering only the palm, the middle finger and the thumb. Includes the determination of:
    - A. Initial position of  $P_{T-C}$  (Heuristic method).
    - B. Inverse kinematics of middle finger and palm.
    - C. Orientation of the wrist.
    - D. Inverse kinematics of thumb.
    - E. Orientation of thumb (angles  $\Theta_{T-7}$  and  $\Theta_{T-8}$ )
    - F. Angles in fingertips of middle finger and thumb (angles  $\Theta_{F-11}$  and  $\Theta_{F-12}$ )
    - G. Position of hand-base  $P_{H-B}$ .
    - H. Configuration of Index finger and ring finger.

- 1.2. Application of a guided search for the 'optimum' position of  $P_{T-C}$ . Includes:
  - I. Definition of a quality measure.
  - J. Search for an optimal configuration.
2. Analysis of the position of the wrist and approximation of the palm position with second order surfaces:
  - K. Partition of the width-height space in approximation regions for the  $X$ ,  $Y$  and  $Z$ -coordinate of  $P_{T-C}$ .
  - L. Approximation of the solution space with second order surfaces.

The following subsections deals with each action from A to L particularized, when necessary, for the hand MA-I (additional details about the derivations of equations can be found in [10]).

#### A. Initial position of $P_{T-C}$ .

The point  $P_{T-C}$  fixes the position of the hand. The initial point  $P_{T-C}$  must ensure that the distances  $D_{TC-MS}$  and  $D_{TC-TS}$  do not exceed their maximum possible value. The usefulness of knowing this point in advance is the reduction of the computing time in the subsequent search.

The positions of  $P_{M-O}$  and  $P_{T-O}$  can be determined from the object width, and, from them,  $P_{M-S}$  and  $P_{T-S}$  can be directly obtained (Figure 1). Using a simple heuristic criterion,  $P_{T-C}$  is initially determined by the intersection of two circles of radius  $D_{TC-MS}$  and  $D_{TC-TS}$  centered on  $P_{M-S}$  and  $P_{T-S}$ , respectively (Figure 3). Both circles lie on a plane, orthogonal to the grasp plane, containing  $P_{M-S}$  and  $P_{T-S}$ . It must be noted that this is an initial value for  $P_{T-C}$ , and since the ranges of the joints were not yet considered, it may produce a solution of the inverse kinematics of the fingers with some joints outside the reachable ranges (in particular,  $\theta_{T-7}$  is likely to be out of range).

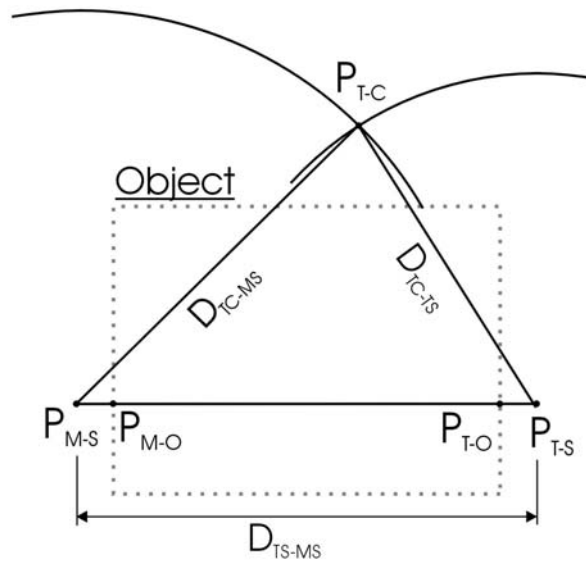


Figure 3. Determination of the initial position of the hand ( $P_{T-C}$ ).



### B. Inverse kinematics of middle finger and palm.

In order to simplify the solution it is considered that for the middle finger  $\Theta_{M-7} = 0$ , while for the other fingers  $\Theta_{T-7}$ ,  $\Theta_{I-7}$  and  $\Theta_{R-7}$  have to be properly determined. The configuration of the palm and the fingers can be determined as a function of the distances between  $P_{T-C}$  and the points  $P_{T-S}$  and  $P_{M-S}$ . The finger joints  $\Theta_{M-8}$ ,  $\Theta_{M-9}$  and  $\Theta_{M-10}$  are coplanar and therefore infinite solutions are possible. In order to determine a solution, it is initially assumed that  $\Theta_{M-8} = \Theta_{M-9} = \Theta_{M-10}$  and the angle  $\Theta_{M-8}$  is solved from the equation of the Euclidean distance between  $P_{T-C}$  and  $P_{M-S}$ . Due to the difficulty in inverting this function, it was approximated with a polynomial function ( $\Theta_{M-8}$  could also be obtained using the Newton's iterative method, however, this results in a series of additional iterative calculations). Using the determined approximated value of  $\Theta_{M-8}$ , the values of  $\Theta_{M-9}$  and  $\Theta_{M-10}$  of the middle finger are calculated by solving them from the finger inverse kinematics using simple trigonometric replacements.

### C. Orientation of the wrist

The next step is the determination of  $\Theta_{H-4}$  and  $\Theta_{H-5}$  according to the current location of  $P_{T-C}$  such that the final position of  $P_{M-S}$  is located according to the object requirements. The known data are  $P_{M-S}$ ,  $P_{T-C}$  and  $P_{M-S} | \Theta_{H-4} = \Theta_{H-5} = 0$ . Then  $\Theta_{H-4}$  and  $\Theta_{H-5}$  are obtained solving the system:

$$(P_{M-S} - P_{T-C}) = R_{\Theta_{H-4}} \cdot R_{\Theta_{H-5}} \cdot (P_{M-S} |_{\Theta_{H-4} = \Theta_{H-5} = 0} - P_{T-C})$$

where  $R_{\Theta_{H-4}}$  and  $R_{\Theta_{H-5}}$  are the rotation matrices for the rotating axis of  $\Theta_{H-4}$  and  $\Theta_{H-5}$ .

### D. Inverse kinematics of thumb

The three angles of the thumb  $\Theta_{T-8}$ ,  $\Theta_{T-9}$  and  $\Theta_{T-10}$  are computed in the same way as in the case of the middle finger to agree with the distance  $D_{TC-TS}$ , but without considering the actual orientation of the thumb with respect to the palm. This orientation will be considered later to recalculate  $\Theta_{T-8}$ , while the obtained  $\Theta_{T-9}$  and  $\Theta_{T-10}$  are final values.

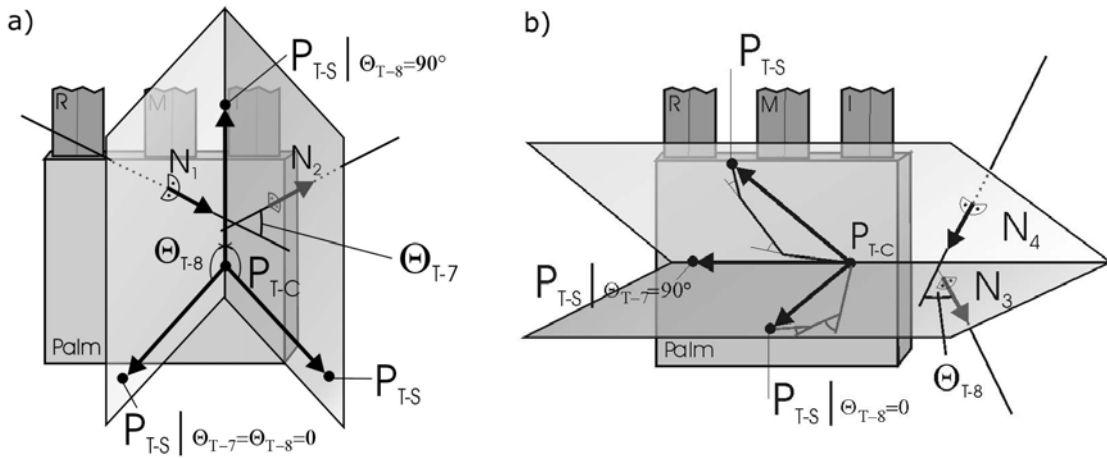


Figure 4. Schematics showing a) the two planes used for the determination of  $\Theta_{T-7}$ ; b) the determination of  $\Theta_{T-8}$ .

### E. Orientation of thumb

The thumb orientation ( $\Theta_{T-7}$  and  $\Theta_{T-8}$ ) is computed as follows.  $\Theta_{T-7}$  is the angle between two auxiliary vectors  $N_1$  and  $N_2$  (Figure 4a):  $N_1$  being the vector normal to the plane swept by the thumb when moving  $\Theta_{T-8}$  for  $\Theta_{T-7} = 0$ , and  $N_2$  the vector normal to the plane defined by  $P_{T-S}$  and the rotation axis of  $\Theta_{T-7}$ . In an analogous way,  $\Theta_{T-8}$  is the angle between another two auxiliary vectors  $N_3$  and  $N_4$  (Figure 4b):  $N_3$  being the vector normal to the plane swept by the thumb when moving  $\Theta_{T-7}$  for  $\Theta_{T-8} = 0$ , and  $N_4$  the vector normal to the plane defined by  $P_{T-S}$  and the rotation axis of  $\Theta_{T-8}$ .

### F. Angles in the fingertips of the middle finger and the thumb

The two virtual articulations describing the actual contact point on the sphere in the fingertips of the middle finger and the thumb ( $\Theta_{T-11}$ ,  $\Theta_{T-12}$ ,  $\Theta_{M-11}$ ,  $\Theta_{M-12}$ ) can now be determined from the position and the orientation of the palm and the positions of the other joints of the middle finger and the thumb. These angles are computed using trigonometric functions and the distances (see Figure 5):  $D_1$  from  $P_{F-O}$  to the plane through  $P_{F-S}$  orthogonal to  $N_5$ ,  $D_2$  from  $P_{F-O}$  to the plane through  $P_{F-S}$  orthogonal to  $N_6$  and  $D_3$  from  $P_{F-O}$  to the plane through  $P_{F-S}$  orthogonal to  $N_7$ . This step is not strictly necessary for the purpose of this work, but it provides information about the fingertip contact point that may be of interest.

### G. Position of hand-base

The computation of the position of the base of the hand,  $P_{H-B}$ , is straightforward from the position and orientation of the palm given by  $P_{T-C}$  (subsection A), and  $\Theta_{H-4}$  and  $\Theta_{H-5}$  (subsection C).

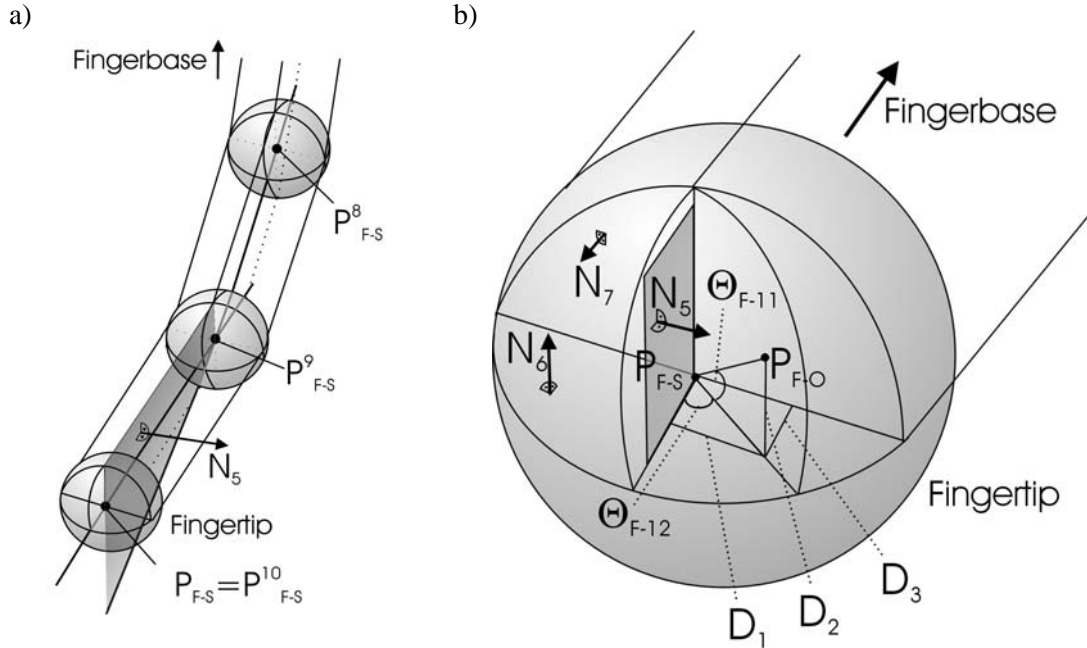


Figure 5. a) Plane formed by the last two phalanges of the finger F. b) Schematics showing the distances  $D_1$ ,  $D_2$  and  $D_3$  necessary for the determination of  $\Theta_{F-11}$  and  $\Theta_{F-12}$ .

### H. Configuration of Index finger and ring finger

Once the configuration of the middle finger and the thumb have been completely determined, the angles  $\Theta_{I-8}$  to  $\Theta_{I-12}$  and  $\Theta_{R-8}$  to  $\Theta_{R-12}$  of the articulations of the index and ring fingers can be determined in a similar way as it was already described for the middle finger. The difference is that now the distances of interest  $D_{IB-IS}$  and  $D_{RB-RS}$  are the distances from the two centres of the spheres  $P_{I-S}$  and  $P_{R-S}$  to the base of the corresponding finger  $P_{I-S}^8$  and  $P_{R-S}^8$ .

### I. Definition of a quality measure

The computation of the overall quality of the arrangement,  $Quality_{\text{Arrangement}}$ , is done as the product of three factors as:

$$Quality_{\text{Arrangement}} = Quality_{\text{Hand}} \cdot Quality_{\text{Fingertips}} \cdot Quality_{\text{Object-Height}}$$

where:  $Quality_{\text{Hand}}$  describes the quality of the hand configuration considering only the articulations of the fingers without the fingertip,  $Quality_{\text{Fingertips}}$  describes the quality of the virtual articulations in the fingertips, and  $Quality_{\text{Object-height}}$  indicates if the hand configuration allow room enough inside it for the predicted height of the object. The following quality functions produced good convergence results.

$Quality_{\text{Hand}}$  is computed as the product of the individual qualities of the hand articulations, obtained each one as: inside the articulation range the quality decreases linearly from 1 at the range centre to 0.5 at the range bound, and outside the range it converges exponentially towards zero (Figure 6a).

$Quality_{\text{Fingertips}}$  is computed as the product of the individual qualities of the virtual articulations in the fingertips, obtained each one as: inside the range the quality-function decreases squarely from 1 at the range centre to 0.5 at the range bound, and outside the range it converges exponentially towards zero.

Small changes in the articulations of the fingers can result in a very large change in the virtual articulations of the fingertips; this is why the quality-function used for the finger joints is slightly modified for the fingertips. The desired effect is that the last phalanges tend to be orthogonal to the object surface.

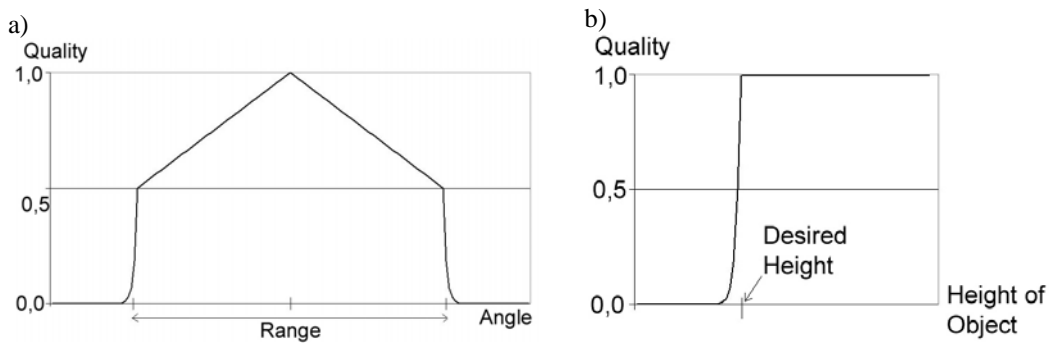


Figure 6. a) Quality-function used for the articulations of the hand.  
 b) Quality-function used for the object height that the configuration of the hand allows.

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For  $Quality_{Object-height}$  it could be sufficient to use a binary variable indicating whether the hand configuration allows the desired object height or not, but the search for an optimum (described below) requires a continuous function to ensure convergence. For this reason,  $Quality_{Object-height}$  has the value 1 if the desired object height can be reached and exponentially decrease towards 0 otherwise (Figure 6b).

#### *J. Search for an optimal configuration*

As it was mentioned before, the heuristic method used to determine the initial value of  $P_{T-C}$  (Subsection IV-A) does not necessarily result in a reachable hand configuration with all the joint angles inside their real ranges. In order to cope with this problem a guided search is applied, which compares the found solution with the surrounding solutions in order to improve the position of the point  $P_{T-C}$  and the resulting hand configuration.

The comparison is done using the quality-functions described in subsection I, with the objective of finding the optimal configuration of the hand with the angles of the articulations close to the middle of their ranges. The search compares the quality of the initial configuration of the hand, placed in the initial position of  $P_{T-C}$ , with solutions found by displacing the point  $P_{T-C}$  around its neighbourhood. The best solution is selected and used as a new starting point for an incremental search until the optimal solution is found.

#### *K. Partition of the width-height space*

Once the optimal configuration of the hand has been determined for a number of pairs [width, height], the X, Y and Z coordinates of position  $P_{T-C}$  (a reference for the hand position) for each case are store as three sets of points [width, height, X], [width, height, Y] and [width, height, Z]. The goal is now the determination of a second order function that approximates each of these three sets of points, i.e.

$$X_{approx} = \alpha_6 \cdot Height^2 + \alpha_5 \cdot Width^2 + \alpha_4 \cdot Height \cdot Width \\ + \alpha_3 \cdot Height + \alpha_2 \cdot Width + \alpha_1$$

for the X coordinate, and equivalent expressions for Y and Z. For this purpose and in order to get a good approximation, the width-height domain is partitioned based on the gradient of the three sets, in such a way that

- The error in the approximated function with respect to the known points becomes smaller than a given threshold;
- The X, Y and Z coordinates of position  $P_{T-C}$  approximated by the second order function allows a kinematics solution for all the considered points [width, height].

The result is a set of regions in the width-height domain, each one with its own second order function for the X, Y and Z coordinates of position  $P_{T-C}$ .

#### *L. Approximation of X, Y and Z coordinates with second order surfaces.*

Within each region obtained in the previous step, the second order surface is estimated from the corresponding sets of points using the least square criterion. This

is achieved by solving the overdetermined system of equations  $B \cdot A = \alpha$ , where  $A$  is the  $n \times 6$  matrix

$$A = \begin{bmatrix} Height_1^2 & & Height_i^2 & & Height_n^2 \\ Width_1^2 & & Width_i^2 & & Width_n^2 \\ Height_1 Width_1 & \dots & Height_i Width_i & \dots & Height_n Width_n \\ Height_1 & & Height_i & & Height_n \\ Width_1 & & Width_i & & Width_n \\ 1 & & 1 & & 1 \end{bmatrix}$$

with  $i$  being the index of the  $n$  pairs [width, height] inside the corresponding region, and  $B$  is the column vector of the known values of each coordinate, e.g.  $[X_1 \dots X_i \dots X_n]$  (idem for  $Y$  and  $Z$ ), and  $\alpha$  is the vector  $[\alpha_6 \dots \alpha_1]^T$  of the second order equation. This operation is easily done using the backslash operator of Matlab. In this way vectors  $\alpha$  describing the second order surface, are easily computed for each coordinate  $X$ ,  $Y$  and  $Z$  in each of the regions in the width-height domain. The second order surfaces were found to be enough for a useful approximation of the position coordinates of the hand.

## v. NUMERICAL RESULTS FOR THE MECHANICAL HAND MA-I

The proposed methodology was implemented in a simulator written in C++ and applied to the mechanical hand MA-I, considering objects in the range of 200x200 mm and samples on a regular mesh with a resolution of 1mmx1mm, i.e. 40000 samples. Figure 7a to Figure 7c shows the  $X$ ,  $Y$  and  $Z$  coordinates of  $P_{T-C}$  of the optimal grasp for the desired height and width of an object (Subsections IV-A to IV-J). The partition of the width-height space divides it into five approximation regions (Figure 7d).

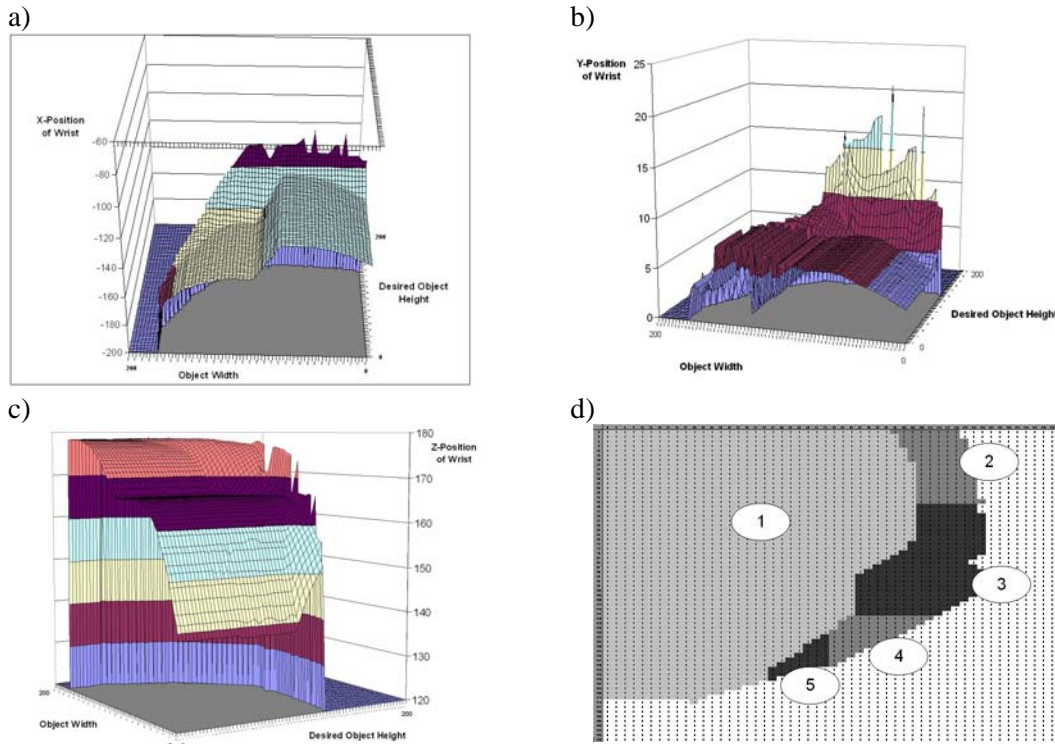


Figure 7. a-c)  $X$ ,  $Y$  and  $Z$ -coordinate of the wrist position.

d) Division of the solution space in five approximation regions.

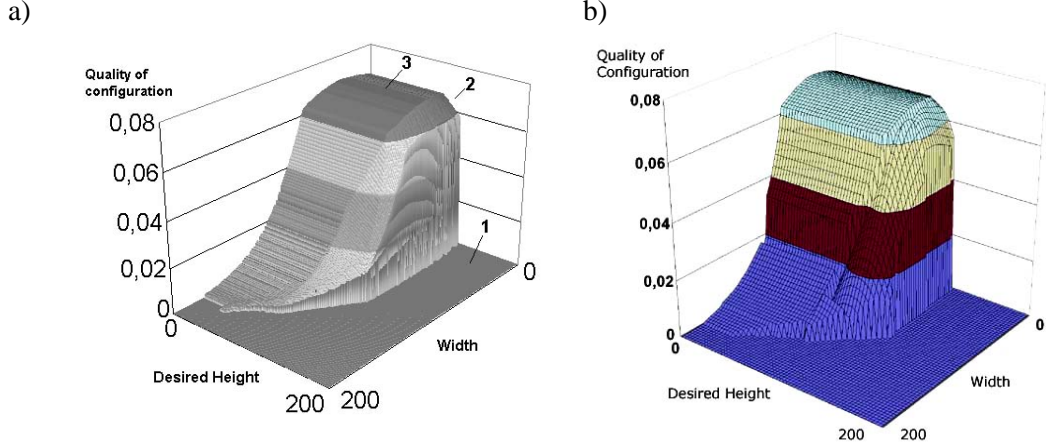


Figure 8. a) Quality of the optimal grasp configurations, (1) no possible solution found, (2) the search is dominated by the desired height of the object, (3) the search of an optimum grasp configuration is dominated by the width of the object; b) Quality of the grasp configurations obtained using the approximation of  $P_{T-C}$ .

Figure 8a shows the  $Quality_{Arrangement}$  of the optimal grasp configuration for the desired heights and widths, and Figure 8b the  $Quality_{Arrangement}$  of the grasps obtained using the approximated  $P_{T-C}$  (Subsections IV-K and IV-L). (Note that these grasps can be obtained on-line with low computational cost). Three parts can be distinguished. The first part (1) shows that in this region no solution is found, i.e. an object with the given width can not be grasped allowing the given height inside the hand. The second part (2) the grasp quality is constrained by fulfilling the desired height of the object and the quality decreases rapidly, meaning that the hand approaches more extreme configurations for large widths or heights. In the third part (3) the grasp quality is constant for a given width regardless of the given height, meaning that the optimal grasp configuration of the hand remains the same for objects with a height smaller than an extreme value, or in other words, the space allowed inside the hand in these optimal grasps is larger than necessary.

As a graphical example, figure Figure 9 shows the obtained optimal configuration of the hand for an object with width = 80mm and height = 100mm; the optimal grasp actually allows an object height of 102.64mm (this case belongs to part 3 in figure Figure 8).

## VI. CONCLUSION

Determining the hand configuration in order to perform a grasp is a hard work due to the high number of involved degrees of freedom and to their distribution. Including any optimization criterion makes it even harder. Dealing with this problem, this paper proposed a method to off-line analyze the wrist position and, from the results, parameterize it as a function of the width and the height of the object. In this way the wrist position information is stored without consuming a lot of memory, and the computation of a quasi-optimal hand configuration can be done when necessary without intensive computation (knowing the object width at the grasp plane and the desired object height “inside” the hand during the grasp, as it is shown in figures 1 and 9).

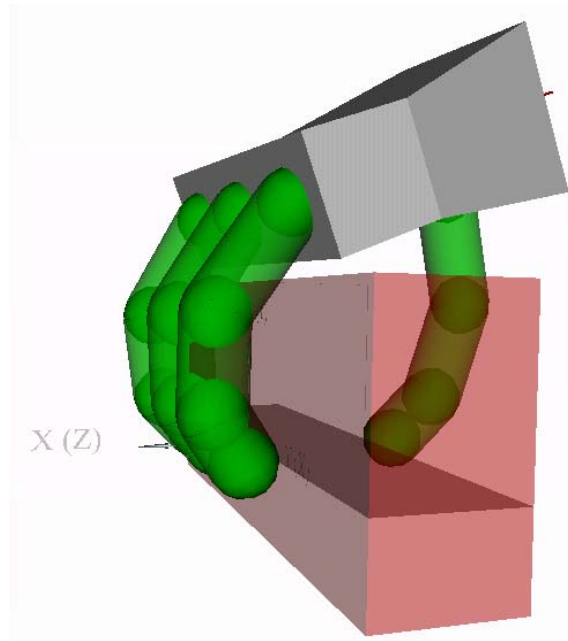


Figure 9. Example of simulation results in VRML of the optimal grasp for an object with width = 80mm and height = 100mm (the optimal grasp allowed an actual height of 102.64mm).

The approach was particularized and implemented for the anthropomorphic mechanical hand MA-I, considering the 16 degrees of freedom of the fingers plus the 8 virtual joints of the fingertips and the 6 degrees of freedom of the wrist. As optimization criterion for the grasp, it was considered that the joints must remain as close as possible to the middle of their ranges. The paper includes numerical results of the implementation.

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